Noise-assisted digital quantum simulation of open systems

José D. Guimarães, James Lim, Mikhail I. Vasilevskiy, Susana F. Huelga, Martin B. Plenio 2022





Overview - Part I

• Use the intrinsic noise of NISQ devices as a resource for quantum computation.



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Overview - Part I

Quantum simulation of Markovian dynamics of open quantum systems;

Use the intrinsic noise of NISQ devices as a resource for quantum computation.



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Overview - Part I

Quantum simulation of Markovian dynamics of open quantum systems;

 The quantum hardware does not need to be changed in order to tune the decoherence rates;

Use the intrinsic noise of NISQ devices as a resource for quantum computation.



Overview - Part II







• Time-evolution in the quantum computer,

Noiseless quantum computer:

 $e^{-i\hat{H}t}|\Psi(0)\rangle \approx \prod_{d=1}^{D} \hat{U}_k(\Delta t)|\Psi(0)\rangle,$

$$\hat{U}_1(\Delta t) = \prod_{j=1}^N e^{-i\hat{H}_j\Delta t}, \quad \Delta t = t/D.$$

 $\hat{H} = \sum_{j=1}^{N} \hat{H}_{j}, \quad \hat{H}_{j} = \alpha_{j} \hat{P}_{j}$



Time-evolution in the quantum computer,

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Noisy quantum computer:

$$\frac{d\hat{\rho}(t)}{dt} = \mathscr{L}[\hat{\rho}(t)]$$

- Markovian noise.

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 $t)] = -i[\hat{H}, \hat{\rho}(t)] + \mathcal{D}_{intrinsic}[\hat{\rho}(t)]$

• Weak noise over a Trotter iteration.



Noise characterization CB + ERErhard, Alexander, et al. *Nature communications* 10.1 (2019): 1-7.

Flammia, Steven T., and Joel J. Wallman. ACM Transactions on Quantum Computing 1.1 (2020): 1-32.

K-qubit stochastic Pauli channel: $\mathscr{E}(\rho) = \sum_{k} \epsilon_{k} \hat{P}_{k} \rho \hat{P}_{k}$





Noise characterization CB + ER

K-qubit stochastic Pauli channel: $\mathscr{E}(\rho) = \sum_{k} \epsilon_{k} \hat{P}_{k} \rho \hat{P}_{k}$

$$\mathsf{K=1:} \quad \mathscr{C}_m^{(1)}(\hat{\rho}) = \epsilon_0 \hat{\rho} + \epsilon_X \hat{X}_m \hat{\rho} \hat{X}_m + \epsilon_Y \hat{Y}_m \hat{\rho} \hat{Y}_m + \epsilon_Z \hat{Y}_m \hat{\rho} \hat{Y}_m + \epsilon_$$

K=2:
$$\mathscr{C}_{m,m+1}^{(2)}(\hat{\rho}) = \mathscr{C}_m^{(1)}(\hat{\rho}) + \mathscr{C}_{m+1}^{(1)}(\hat{\rho})$$

 $+\epsilon_{XX}\hat{X}_m\hat{X}_{m+1}\hat{\rho}\hat{X}_m\hat{X}_{m+1}, +\epsilon_{XY}\hat{X}_m\hat{Y}_{m+1}\hat{\rho}\hat{X}_m\hat{Y}_{m+1} + \cdots$





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No need to characterize errors acting on more than K=2 nearest-neighbour qubits.





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Hashim, Akel, et al. arXiv preprint arXiv:2010.00215 (2020). Wallman, Joel J., and Joseph Emerson. Physical Review A 94.5 (2016): 052325.

Randomized Compiling



 $\hat{Z}_m \hat{\rho} \hat{Z}_m$



Time-evolution in the quantum comp

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Noisy quantum computer:

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$$\hat{H} = \sum_{j=1}^{N} \hat{H}_j, \quad \hat{H}_j = \alpha$$

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 $\frac{d\hat{\rho}(t)}{dt} = \mathscr{L}[\hat{\rho}(t)] = -i[\hat{H}, \hat{\rho}(t)] + \mathscr{D}_{\text{stochastic}}[\hat{\rho}(t)]$





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$$_{\text{chastic}}[\hat{\rho}(t)] = \sum_{k=0}^{4^{K}-1} \gamma_{k} \left(\hat{P}_{k} \hat{\rho}(t) \hat{P}_{k} - \hat{\rho}(t) \right), \quad \gamma_{k} = \epsilon_{k} / \Delta t$$

$$\mathsf{K=2}$$





$$\hat{H} = -\sum_{m=1}^{n} \frac{E_m}{2} \hat{Z}_m + \sum_{m=1}^{n-1} \frac{J_{m,m+1}}{2} (\hat{X}_m \hat{X}_{m+1} + \hat{Y}_m \hat{Y}_{m+1})$$













0.6

0.8



Control of noise



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$$\frac{d\hat{\rho}(t)}{dt} = \mathscr{L}[\hat{\rho}(t)] = -i[\hat{H}, \hat{\rho}(t)] + \mathscr{D}_{\text{stochastic}}[\hat{\rho}(t)]$$
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How can we control γ_k in the digital simulation without changing the hardware components?

Control of noise

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$$\textbf{Target decoherence rates } \Gamma_{k}$$
$$\mathscr{D}_{\text{stochastic}}^{(\text{controlled})}[\hat{\rho}(t)] = \sum_{k=0}^{4^{K}-1} \Gamma_{k} \left(\hat{P}_{k}\hat{\rho}(t)\hat{P}_{k} - \hat{\rho}(t)\right), \quad \Gamma_{k} = \epsilon_{k}$$

$$-\hat{\rho}(t)\Big), \quad \Gamma_{k} = \epsilon_{k}(1-r_{k})/\Delta t \,.$$

$$r_{k} \in [0,1]$$
Probabilistic Error Cancelation

Endo, Suguru, Simon C. Benjamin, and Ying Li. *Physical Review X* 8.3 (2018): 031027. Sun, Jinzhao, et al. *Physical Review Applied* 15.3 (2021): 034026.

 Probabilistically cancels Markovian noise to first order by inverting the noise channel.

Characterization of the noise channel acting in t

Inversion of the characterized stochastic Pauli c

The circuit:
$$\mathscr{E}(\rho) = \epsilon_0 \rho + \sum_{k=1} \epsilon_k P_k \rho P_k$$
 CPTP map
channel: $\mathscr{E}^{-1}(\rho) = \epsilon'_0 \rho - \sum_{k=1} \epsilon'_k \rho P_k$ Non-CP map

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$\mathscr{E}^{-1}(\rho) = \sum q_k \mathscr{P}_k(\rho) = \sum q_k P_k \rho P_k, \quad q_k = \pm \epsilon'_k$ k=0k=0

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$$\mathcal{E}^{-1} = C \sum_{k=0} p_k \operatorname{sign}(q_k) \mathcal{P}_k$$

$$p_k = \epsilon'_k / C$$
$$C = \sum_{k=0}^{\infty} |q_k|$$

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Probability of sampling P_k

$$p_k = \epsilon'_k / C$$

$$C = \sum_{k} |q_k|$$

k=0

k=0

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$$p_{k} = \epsilon_{k}^{\prime}/C$$

$$C = \sum_{k=0}^{\infty} |q_{k}|$$
Mitigation cost

Implementation

$$C = \sum_{k=0} |q_k| = 1 + 2r \sum_{k=1} \epsilon_k$$
(a) 1.0
(b) 0.8
(c) 0.6
(c) 0.06

Partial Probabilistic Error Cancelation Mitigation Cost

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For a Trotter-type circuit:

 $C_{tot}^2 \sim e^{\lambda n D \epsilon_r}$

$$\epsilon_r = \sum_{k=1}^{r} \epsilon'_k = \sum_{k=1}^{r} r_k \epsilon_k$$

Decoherence rate control scheme

How can we control γ_k in the digital simulation without changing the hardware components?

Target decoherence rates Γ_k :

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$$\Gamma_k = \epsilon_k (1 - r_k) / \Delta t_k$$

 $C_{\rm tot}^2 \sim e^{\lambda n D \epsilon_r} \lesssim M_{\rm max}^{(\rm NISQ)}$

Target decoherence rates Γ_k : $\Gamma_k = \epsilon_k (1-r_k)/\Delta t$

$$\epsilon \lesssim \frac{\Gamma t}{D} + \frac{\ln(M_{\max}^{(NISQ)})}{2\lambda n D^2}$$

$$\epsilon = \sum_{k=1} \epsilon_k \qquad \Gamma = \sum_{k=1} \Gamma_k$$

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Target decoherence rates Γ_k : $\Gamma_k = \epsilon_k (1 - r_k) / \Delta t$

$$\epsilon \lesssim \frac{\Gamma t}{D} + \frac{\ln(M_{\max}^{(\text{NISQ})})}{2\lambda n D^2} \quad \clubsuit \quad \epsilon \lesssim O\left(\frac{\varepsilon_{\text{Trot}}\Gamma}{n^d J(J+E)t} + \frac{\varepsilon_{\text{Trot}}^2 \ln(M_{\max}^{(\text{NISQ})})}{2\lambda n^{2d+1} J^2 (J+E)^2 t^4}\right)$$

 $\epsilon = \sum_{k=1} \epsilon_k \qquad \Gamma = \sum_{k=1} \Gamma_k$

Applications: NISQ computers and quantum analog simulations.

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Note:

Technique is not restricted only to stochastic Pauli channels

- Applications: NISQ computers and quantum analog simulations.
- The quantum hardware does not need to be changed in order to tune the noise rate;
- Open quantum systems coupled to Markovian environments via stochastic Pauli noise channels with medium- or large-decoherence rates.
- Ongoing work: Generalization of this technique to simulate non-perturbative dynamics of open quantum systems.

Thank you for your attention!

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